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Internal down quark's flavour changing contribution to the effective tcZ vertex

Surath Kumar Biswas, V.P. Gautam

Theoretical Physics Department, Indian Association for the Cultivation of Science, Jadavapur, Calcutta, 700032, India
(e-mail: tpvpg@iacs.ernet.in)

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Abstract. The effective tcZ vertex may be influenced by tree $dd'Z$ vertex formed by a mixing with heavy exotic isosinglet down type quarks. To study that the electroweak penguin diagrams involving one insertion of the $dd'Z$ vertex have been considered and we have calculated the contribution arising out of those diagrams using the Fourth Generation CKM matrix elements; also the applicability of the generalized *GIM* mechanism is considered. The additional effects of the heavy isosinglets are compared with the effects of exotic heavy isodoublets appearing in multi-generational models. We see that in the effective vertex amplitude, the down flavour changing contribution interferes constructively with the one loop penguin diagrams and experimental data may put constraint on the fourth generation down quark mass.

1 Introduction

It is admitted fact that due to the unitarity of the CKM (Cabbibo Kobayashi Maskawa) matrix [1] in the Flavour Changing Neutral Current (FCNC) processes in the Standard Model (SM) [2] the leading order mass independent term is strongly suppressed by GIM [3] cancellation mechanism and this is experimentally confirmed and this paves the way for investigating the new sources of FCNC. So the study of virtual effects opened hydraheaded windows on electroweak symmetry breaking and physics beyond the SM. The examination of these indirect effects of new physics in higher order processes yields a complimentary approach to the search for direct production of new particles at high energy colliders.

On FCNC of radiative b decays a review by Greub et al. on the next-to-leading logarithmic results has appeared in proceedings of a recent Symposium [4]. Chetyrkin et al. [5] have obtained the results for three loop anomalous dimensions while analyzing $B \to X_s\gamma$ decay and report the branching ratio $\mathcal{B}(B \to X_s\gamma)$ to be $(3.28 \pm 0.33) \times 10^{-4}$. The predictions of the Standard Model are in conformity with the CLEO data at 2σ level. The new results opened the scope for investigations in various classes of models, namely: Anomalous Top-Quark Couplings [6], Anomalous Trilinear Gaube Couplings [7], Fourth Generation [8], Two-Higgs-Doublet Model [9], Three-Higgs-Doublet Model [10], Supersymmetry [11], Extended Technicolour [12], Leptoquarks [13], Left-Right Symmetric Models [14].

Apart from these models, in the line of investigation conducted recently in LEP we are contemplating the existence of a new U(1) gauge boson coupling predominantly to the third family and it may have the consequence of enhancing the b quark decay modes [15]. The sources of FCNC may also be coming from (i) the ratios between the masses of fermions involved in the FC transitions, or (ii) some new mass scale of the order of electroweak breaking scale or it may be larger than this where it may arise from mixing between the light fermions and new heavy states with non-standard $SU(2)_L$ assignments [16–19] or from Multi-Higgs Doublets Model without natural flavour conservation [20, 21] or by horizontal symmetries [22] in fermion mass hierarchy. Due to the fact that the fermion masses are small the effects are naturally suppressed. But now the appearance of the top quark into the picture with heavy mass of 180 GeV has changed the scenario abruptly where the FC transition involves the t quark. Of late investigation is going on at the phenomenological level [23–25] and also for model building [20, 21].

Here we try to find out to what extent the effective tcZ vertex is modified by inserting a tree level $dd'Z$ FCNC vertex, assuming mixing between d, s, b quarks and new isosinglet heavy states of charge $-\frac{1}{3}$.

2 Calculation of tcZ vertex without FCNC

One loop diagram for the tcZ vertex is given in Fig. 1. The blob in the diagram represents the self-energy part of the $t \leftrightarrow c$ transition. The one loop diagrams for this transition are shown in Fig. 2. The induced tcZ vertex takes the form

$$A^{(i)}_{Z_\mu} = \frac{g^3}{(4\pi)^2 c} U^*_{cj} U_{tj} (\bar{c}_L \gamma_\mu t_L) A^i \tag{1}$$

where $i = a, b, \ldots, h$. We now write the one loop contribution to the $t\bar{c}Z$ coupling by a direct summation of Feynman graphs defined in (1) for each of the diagrams in

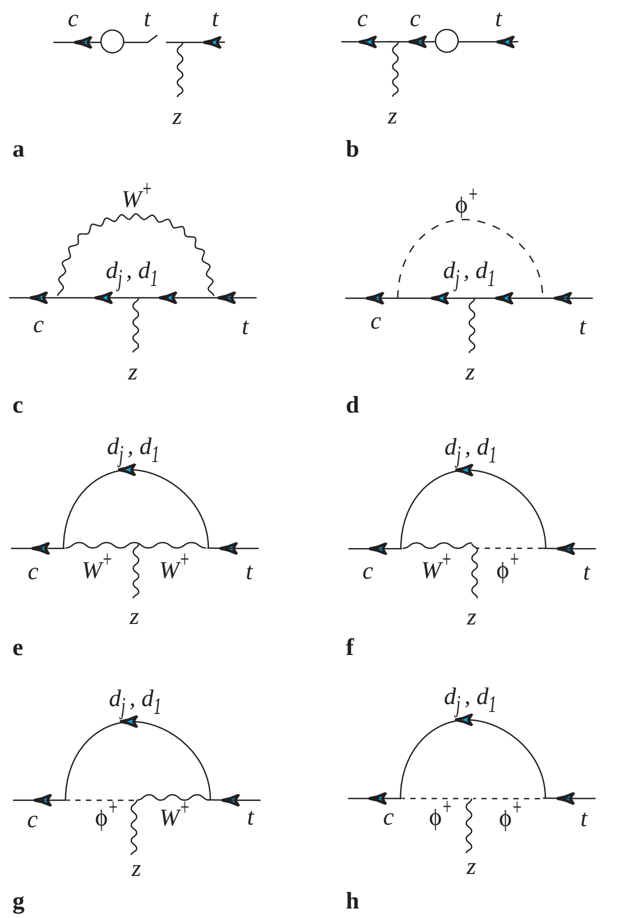

Fig. 1. The one-loop diagrams contributing to the induced tcZ vertex. The blob in the diagram **a** and **b** represents self-energy part of the $t \leftrightarrow c$ transition

Fig. 1 as follows:

$$A^{(a+b)} = -\left[\frac{1}{2}(Q-1)\sin^2\theta_W + \frac{1}{4}\right] \times \left[\frac{x_j^2}{(x_j-1)^2}\ln x_j - \frac{x_j}{x_j-1} - x_j f_1(x_j)\right] -(x_j \to x_1) \,, \tag{2}$$

$$A^{(c)} = \left(\frac{1}{2}Q\sin^2\theta_W - \frac{1}{4}\right)\frac{x_j^2}{(x_j-1)^2}\ln x_j + \frac{1}{2}\frac{x_j}{(x_j-1)^2}\ln x_j - \left(\frac{1}{2}Q\sin^2\theta_W + \frac{1}{4}\right)\frac{x_j}{x_j-1} - (x_j \to x_1) \,, \tag{3}$$

$$A^{(d)} = -\frac{1}{2}Q\sin^2\theta_W\left(1-\frac{2}{n}\right)x_j f_2(x_j) + \frac{1}{4}(Q\sin^2\theta_W - 1) \times \left[\frac{x_j^2}{(x_j-1)^2}\ln x_j - x_j - \frac{x_j}{x_j-1}\right] -(x_j \to x_1) \,, \tag{4}$$

$$A^{(e)} = \frac{3}{2}(1-\sin^2\theta_W)\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j - \frac{1}{x_j-1}\right] -(x_j \to x_1) \,, \tag{5}$$

$$A^{(f+g)} = \sin^2\theta_W\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j - \frac{x_j}{x_j-1}\right] -(x_j \to x_1) \,, \tag{6}$$

$$A^{(h)} = \left(\frac{1}{2} - \sin^2\theta_W\right) \left\{\frac{1}{4}\left[\frac{x_j^2}{(x_j-1)^2}\ln x_j - \frac{x_j}{x_j-1}\right] - \frac{1}{n}x_j f_2(x_j)\right\} -(x_j \to x_1) \,. \tag{7}$$

where $x_j = \frac{m_j^2}{m_W^2}$, m_j is the mass of d_j quark, and $Q = -\frac{1}{3}$, the charge fo the internal down type quarks; we assume m_1 to be very small as compared to others.

$$f_1(x) = -\frac{1}{n-1} - \frac{1}{2}(-\gamma_E + \ln(4\pi) - \ln m_W^2) + \frac{3}{4} - \frac{1}{2}\left(\frac{x^2}{(x-1)^2}\ln x - \frac{1}{x-1}\right) \,, \tag{8}$$

$$f_2(x) = -\frac{2}{n-4} - \gamma_E + \ln(4\pi) - \ln m_W^2 + 1 - \frac{x}{(x-1)}\ln x \,. \tag{9}$$

γ_E is Euler-Mascheroni constant. Thus to find $\mathcal{A}_j^{tcZ}$ for the d_1 and d_j the internal quarks in the loop we sum all the A^i's and we get the R_ξ gauge as

$$\begin{aligned}\mathcal{A}^{tcZ}(x_j) &\equiv \sum_{i=1}^{h} A^i \\ &= \frac{1}{4}x_j - \frac{3}{8}\frac{1}{x_j-1} + \frac{3}{8}\frac{2x_j^2 - X_j}{(x_j-1)^2}\ln x_j \\ &\quad + \frac{1}{\xi x_j - 1}\left(\frac{3}{4}\frac{1}{x_j-1} + \frac{1}{8}\frac{1}{\xi x_j - 1}\right)x_j \ln x_j \\ &\quad - \frac{1}{8}\frac{1}{\xi}\frac{1}{\xi x_j - 1}\left[\left(\frac{5\xi+1}{\xi-1} - \frac{1}{\xi x_j - 1}\right)\ln\xi + 1\right]\end{aligned} \tag{10}$$

We see that the final result is independent of the internal quark charge and pole at $n = 4$ for (2), (4) and (7) are eliminated when summed. We further note that $A^{(f+g)}$ does not contain any divergent integral and those in $A^{(c)}$ and $A^{(e)}$ are killed by *GIM* cancellation mechanism due to unitarity of the CKM matrix. And so in 't Hooft-Feynman gauge where $\xi = 1$, one gets

$$\mathcal{A}_{eff}^{without\ FC} = \frac{g^3}{(4\pi)^2 \cos\theta_W} (\bar{c}_L \gamma_\mu t_L) \left(\sum_{d=d,s,b} \xi_d \mathcal{A}^{tcZ}(x_d) \right) \tag{11}$$

with $\xi_i = V_{ci}^* V_{ti}$, and $x_d = \frac{m_d^2}{m_W^2}$. Now the above expression is not gauge invariant by itself. To make it gauge invariant one has to add box diagrams for the processes $t \to c l \bar{l}$, with $l = \nu, l^{\pm}$.

Introducing the Inami and Lim function [26]

$$F(x) = \frac{5}{2} \left[\frac{1}{x-1} - \frac{x \ln x}{(x-1)^2} \right] \tag{12}$$

we have physical gauge invariant quantity for the decay amplitude

$$\mathcal{A}_{eff}^{without\ FC} = \frac{g^3}{(4\pi)^2 \cos\theta_W} \left(\sum_{d=d,s,b} \xi_d (\mathcal{A}^{tcZ}(x_d) + F(x_d)) \right) (\bar{c}_L \gamma_\mu t_L)\ . \tag{13}$$

3 General formulation

We assume the existence of n new $Q = -1/3$ isosinglet L-handed quarks D_L^0. They can appear in vector like multipletes D_L^0, D_R^0 and they are mixed with unknown down type quarks d_L^0, d_R^0. The number n of $D_L^0 - D_R^0$ pairs is not that relevant for our formulation in general and so we keep it unspecified for the present. D_R^0 and d_R^0, being both colour triplet $Q = -1/3$ isosinglet states, have the same gauge quantum numbers, and then their coupling to the gauge bosons are unaffected by the mixing. This is not the case for the L-chirality states. The vector

$$\mathcal{D}_{dL}^0 = \begin{pmatrix} d^0 \\ D^0 \end{pmatrix}_L$$

of the doublet (d^0) and the singlet (D^0) gauge eigenstates is related to the corresponding vector of the "light" (d) and "heavy exotic" (D) mass eigenstates

$$\mathcal{D}_{dL} = \begin{pmatrix} d \\ D \end{pmatrix}_L$$

through a unitary matrix II (which is a 3×3 matrix) such that

$$\begin{pmatrix} d^0 \\ D^0 \end{pmatrix}_L = \prod \begin{pmatrix} d \\ D \end{pmatrix} \tag{14}$$

$$\prod = \begin{bmatrix} P & Q \\ R & S \end{bmatrix} \tag{15}$$

here $D = [b_1, b_2, \ldots, b_n]^T$ and $d = [d, s, b]^T$. Yet $\prod$ is unitary P and R are not themselves unitary. In the weak basis, the charged fermion neutral current shall contain $P^\dagger P$ and $R^\dagger R$ which are not necessarily diagonal and thus the mixing in general induces FCNC's among the light paricles. In order to avoid this problem, the assumption [18] which is made is that each ordinary left- and right-handed fermion mix with its own exotic partner. In this case $P^\dagger P$ and $R^\dagger R$ are diagonal and thus eliminating FCNC's. With this assumption we can write $(P_a^\dagger P_a)_{ij} = (c_a^i)^2 \delta_{ij}$, $(R_a^\dagger R_a)_{ij} = (s_a^i)^2 \delta_{ij}$, a = Left, Right. Here $(s_a^i)^2 \equiv 1 - (c_a^i)^2 \equiv \sin^2\theta_a^i$, where $\theta^i_{Left(Right)}$ is the mixing angle in the ith Left-handed(Right-handed) ordinary fermion and its exotic partner. The unitary of $\prod$ implies $\prod^\dagger \prod = \prod \prod^\dagger$ = Unit Matrix, and so

$$P^\dagger P + R^\dagger R = P P^\dagger + Q Q^\dagger \equiv I_{3\times3} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{16}$$

We now introduce a unitary matrix Δ (a 3×3 matrix) for the L-handed up type quarks, so that

$$u_L^0 = \Delta u_L \qquad \Delta\Delta^\dagger = \Delta^\dagger \Delta = I_{3\times3} \tag{17}$$

We now introduce a $(3+n) \times 3$ matrix[1] X given by

$$X = \begin{pmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 \end{pmatrix} \tag{18}$$

and with the help of (1) we write the Charged Current (CC) coupled to the $W^{\pm}$ bosons as

$$\frac{1}{2} J_\mu^W = \overline{u_L^0} \gamma_\mu X \mathcal{D}_{dL}^0 = \overline{u_L} \gamma_\mu \Delta^\dagger X \prod \mathcal{D}_{dL} \tag{19}$$

From above we see that we can define a $3 \times (3+n)$ mixing matrix V given by

$$V = \Delta^\dagger X \prod \equiv \left[V_{d_{3\times3}}\ V_{D_{3\times n}} \right] = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub_1} & \ldots & V_{ub_n} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb_1} & \ldots & V_{cb_n} \\ V_{td} & V_{ts} & V_{tb} & V_{tb_1} & \ldots & V_{tb_n} \end{bmatrix} \tag{20}$$

We identify $V_{d_{3\times3}}$ as the usual 3×3 Cabbibo Kobayashi Maskawa (CKM) matrix [1] for the light states d and we see that it is not unitary. We see from (16), (17) and (20)

$$V V^\dagger = \Delta^\dagger X^\dagger \prod \prod^\dagger X \Delta = I_{3\times3} \tag{21}$$

[1] Such choice is made due to the arrangement of the CKM matrix; where the insertion of the exotic down type quarks can be made by augmenting in columns. While the insertion of the up type will require augmenting in rows.

Thus V is analogous to the unitary CKM matrix.

Now for θ_W 'Weinberg weak mixing angle', $s = \sin\theta_W$, $c = \cos\theta_W$, and we define a $(3+n)\times(3+n)$ matrix

$$I_3 = X^\dagger \times X \equiv \begin{pmatrix} I_{3\times 3} & 0 \\ 0 & 0 \end{pmatrix} \tag{22}$$

the projector operator acting on the L-handed $\frac{1}{2}$ isospin doublet states; and we can write the neutral current (NC) coupled to the Z boson in terms of the mass eigenstates, as

$$\frac{1}{2}J^W_\mu = -\frac{1}{2}\overline{\mathcal{D}_{dL}}\gamma_\mu \Delta^\dagger I_3 \Delta \mathcal{D}_{dL} - s^2 \overline{\mathcal{D}_d}\gamma_\mu E \mathcal{D}_d \tag{23}$$

In (23) the second term remains flavour diagonal since the matrix of electric charge E is proportional to the Identitiy i.e., $E = \frac{-1}{3}\times$ Identity. But when we consider the current matrix, the isospin part $\frac{1}{2}I_3$ is not proportional to the Identity, and therefore the corresponding isospin couplings are Flavour Changing.

Let us now define the Neutral Current mixing matrix as $\mathcal{N} = \prod^\dagger I_3 \prod$. We see that $\mathcal{N}$ is not unitary, and it is $(3+n)\times(3+n)$ matrix.

But from (17) and (20) we get

$$\begin{aligned} \mathcal{N} &= V^\dagger V \ , \\ \mathcal{N}^\dagger \mathcal{N} &= \mathcal{N}^2 = \mathcal{N} \ , \text{ and} \\ V\mathcal{N} &= V \end{aligned} \tag{24}$$

we see that $\mathcal{N}$ is idempotent since from the definition of $\mathcal{N}$ we can interpret $\mathcal{N}$ as the projection operator on the L-doublets written on the basis of mass eigenstates.

On actual calculation we see that $\mathcal{N}_{dd} = 0.9796$, and $\mathcal{N}_{ss} = 0.9953$ all ≈ 1. Now we may note further that $\mathcal{N}_{dd} \approx \mathcal{N}_{ss} \approx 1$ because of the experimental bounds on the left handed down and strange quarks which are flavour diagonal and we note again that as

$$\sum_{a=1}^{n} \mathcal{N}_{aa} = Tr(V^\dagger V) = 3 \tag{25}$$

we get $\sum_{a=3}^{n} \mathcal{N}_{aa} \sim 1$. Further we may note that $\mathcal{N}$ is not symmetric, $\mathcal{N}_{dd'} \neq \mathcal{N}_{d'd}$.

Now $VV^\dagger = I_{3\times 3}$, and $V\mathcal{N} = V$ implies $V\mathcal{N}V^\dagger = VV^\dagger = I_{3\times 3}$ and so all the mass independent terms in the new penguin diagrams which carry structure $V\mathcal{N}V^\dagger$ are cancelled off in spite of the presence of the FC couplings. Actually we can write from (21) and (24)

$$\sum_{jk} V^*_{ck}(\delta_{jk} - \mathcal{N}_{jk})V_{tj} = 0 \tag{26}$$

Now following [27] we get the usual SM L-handed and R-handed chiral couplings of down type quarks as

$$\begin{aligned} g^d_L &= -\frac{1}{2} + \frac{1}{3}s^2, \ and \\ g^d_R &= \frac{1}{3}s^2 \end{aligned} \tag{27}$$

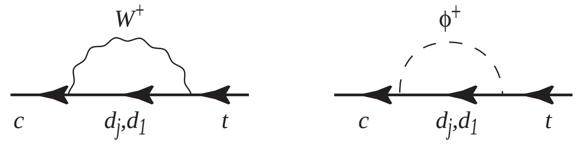

Fig. 2. The one-loop contributing to the self-energy part of the $t \leftrightarrow c$ transition; the *blob* in Fig. 1

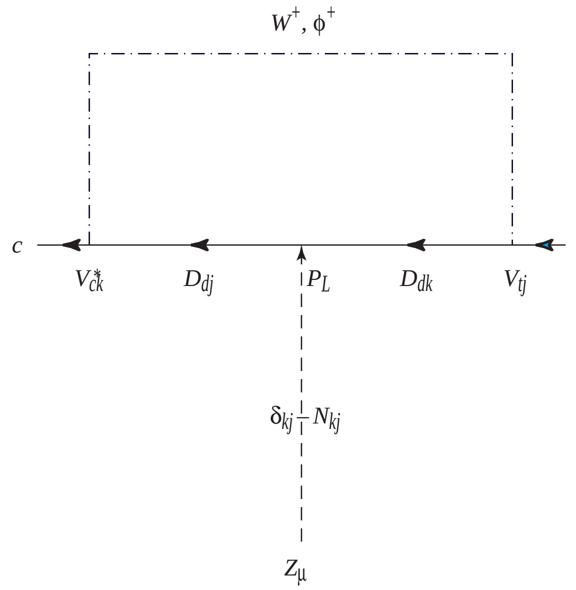

Fig. 3. Electroweak penguin diagrams with W boson and scalar ϕ which include the flavour changing vertex $\mathcal{N}_{kj,j\neq k}$. The relevant mixing matrices appearing at the vertices are shown explicitly, and $P_L = \frac{1}{2}(1-\gamma_5)$ is the L-handed chiral projector. The quarks are denoted by $\rightarrow$; W, ϕ are denoted by $- \cdot - \cdot \rightarrow$ and Z is denoted by $- - -$

Table 1. Function against masses in GeV $x = \frac{mass^2}{m^2 W}$

Function	0.015	0.15	4.5
$\mathcal{A}^{tcZ}(x)$	1.25	1.24998	1.2456
$F(x)$	−2.50	−2.4999	−2.46228
Total	−1.25	−1.2499	−1.2167

From (23) it is evident that L-handed down quark coupling is changed for mixing with the new isosinglets and introduces an FC term. For the sake of generality we write $\mathcal{D}_{d_i}\mathcal{D}_{d_j}Z$ coupling as

$$\begin{aligned} g^{ij}_L &= -\frac{1}{2}\mathcal{N}_{ij} + \frac{1}{3}s^2\delta_{ij} \\ &= g^d_L \delta_{ij} + \frac{1}{2}(\delta_{ij} - \mathcal{N}_{ij}) \\ &Where\ i,j = d,s,b,b_1,b_2,\ldots,b_n \ . \end{aligned} \tag{28}$$

The term g^d_L represents the extension of SM to $3+n$ L-handed doublets with no tree level FCNC and the term $\frac{1}{2}(\delta_{ij} - \mathcal{N}_{ij})$ shows that the new n states are isosinglets.

The first term gives us scope to compare results for the isosinglets case with those of a multigenerational model.

Further, from (28) we see that the calculation of effective tcZ vertex in presence of the tree level FC couplings can be done by calculating SM contribution [27] extended to $3+n$ generations and by computing two additional diagrams given in Fig. 3 which arise from the second term in (28).

4 Calculation of FC amplitude by inclusion of exotic down quarks

The amplitude for the sum of the one loop penguin diagrams which do not contain any insertion for the FC couplings in the 't Hooft-Feynman gauge is given in (11) and the physical gauge invariant quantity for the decay amplitude is given in (13).

Now we turn our attention to the second term in (28), namely: $\frac{1}{2}(\delta_{ij} - \mathcal{N}_{ij})$. We have considered new states to be isosinglet so there shall be no extra contributions from the FC couplings. The diagram corresponding to this term as stated earlier is given in Fig. 3. The loops at W boson or the scalar ϕ is logarithmically divergent. But the Chiral projection operator $P_L = \frac{1}{2}(1-\gamma_5)$ reduces the degree of divergence by a factor 2 thus the divergence is eliminated.

From (26) we see that when we sum over all the d and D, all the terms independent of d and D masses and in particular the poles at $n = 4$ are wiped out like the GIM cancellation law and thus leading to finite contribution from the diagram involving W boson loop.

Now we write the amplitude obtained from the second term in (28) as

$$\begin{aligned}\mathcal{A}_{eff}^{with\ FC} &= \mathcal{A}_W + \mathcal{A}_\phi \\ &= \frac{g^3}{(4\pi)^2\cos\theta_W} \\ &\quad \left(\sum_{k,j} V_{ck}^*(\delta_{kj} - \mathcal{N}_{kj})V_{tj}F(x_k, x_j)\right)(\bar{c}_L\gamma_\mu t_L)\end{aligned} \tag{29}$$

where

$$F(x,y) = \frac{1}{4(x-y)}\left(\frac{y-1}{x-1}x^2\ln x - \frac{x-1}{y-1}y^2\ln y\right) \tag{30}$$

Where k, j runs over $d, s, b, b_1, b_2, \ldots, b_n$. We may note that (29) is also gauge invariant.

Amplitude for the effective tcZ vertex now can be written as

$$\begin{aligned}\mathcal{A}^{tcZ} &= \mathcal{A}_{eff}^{withoutFC} + \mathcal{A}_{eff}^{withFC} \\ &= \frac{g^3}{(4\pi)^2\cos\theta_W}\left(\sum_{d=d,s,b}\xi_d(\mathcal{A}^{tcZ}(x_d) + F(x_d))\right) \\ &\quad (\bar{c}_L\gamma_\mu t_L) \\ &\quad + \frac{g^3}{(4\pi)^2\cos\theta_W} \\ &\quad \left(\sum_{d,d'} V_{cd}^*(\delta_{dd'} - \mathcal{N}_{dd'}(V_{td'}F(x_d, x_{d'})))\right) \\ &\quad (\bar{c}_L\gamma_\mu t_L)\end{aligned} \tag{31}$$

Now we see that as $x \to y$,

$$\begin{aligned}F(x,y) &\to \frac{x}{4} - \frac{x\ln x}{2(x-1)} \\ &= F_1(x)\end{aligned} \tag{32}$$

which is included in $\mathcal{A}^{tcZ}(x)$.

Then we can write the amplitude as

$$\begin{aligned}\mathcal{A}^{tcZ} &= \frac{g^3}{(4\pi)^2\cos\theta_W} \\ &\quad \left(\sum_{d=d,s,b}\xi_d(\mathcal{A}^{tcZ}(x_d) + F(x_d))\right)(\bar{c}_L\gamma_\mu t_L) \\ &\quad + \frac{g^3}{(4\pi)^2\cos\theta_W} \\ &\quad \left(\sum_{d}^{b_n} V_{cd}^*(1 - \mathcal{N}_{dd})V_{td}F_1(x_d)\right)(\bar{c}_L\gamma_\mu t_L) \\ &\quad - \frac{g^3}{(4\pi)^2\cos\theta_W} \\ &\quad \left(\sum_{d,d'}^{d\neq d'}(V_{cd}^*\mathcal{N}_{dd'}V_{td'})F(x_d, x_{d'})\right)(\bar{c}_L\gamma_\mu t_L)\end{aligned} \tag{33}$$

Thus from the note below (24) we see that for the second sum in the above expression first two terms are not contributing much and contributions are coming from the bottom quark and $b_1, b_2, \ldots$.

5 Results and discussion

We first turn our attention to the matrices V and $\mathcal{N}$. As stated earlier V_L is not unitary and following [18] the elements can be written as

$$V_{L\,ij} = c_L^{u_i} c_L^{d_j} \hat{V}_{L\,ij} \tag{34}$$

where $\hat{V}_L$ is usual unitary CKM matrix. The values of $c_L^{u_i}$ and $c_L^{d_j}$ are calculated from the values of s's collected from [28] given below:

$$\begin{aligned}(s_L^d)^2 &= 0.0023, \quad (s_R^d)^2 = 0.019 \\ (s_L^s)^2 &= 0.0036, \quad (s_R^s)^2 = 0.021 \\ (s_L^c)^2 &= 0.0042, \quad (s_R^c)^2 = 0.010 \\ (s_L^b)^2 &= 0.0020, \quad (s_R^b)^2 = 0.010\end{aligned}$$

Now we calculate the amplitude $\mathcal{A}^{tcZ}$ given in (33) in units of $\frac{g^3}{(4\pi)^2\cos\theta_W}(\bar{c}_L\gamma_\mu t_L)$ term by term. We have used the CKM matrix elements from the Fourth Generation calculated in [29].

We take $m_d = 0.015\,\text{GeV}$, $m_s = 0.15\,\text{GeV}$, $m_b = 4.5\,\text{GeV}$ and $m_W = 80.22\,\text{GeV}$. The first term of (33) is without FC. The values of the functions $\mathcal{A}^{tcZ}(x)$ and $F(x)$

Table 2. Function against masses in GeV $x = \frac{mass^2}{m^2W}$

Function	0.015	0.15	4.5	50	100	150	20
$F_1(x)$	-2.8×10^{-7}	-2.1×10^{-5}	-8.3×10^{-3}	-0.203	-0.229	-2.479	$+0.46$

Table 3. Function $F(x_d, x_{d'})$ for $d \neq d'$ $x = \frac{mass^2}{m^2W}$

Quark mixing$^{\rightarrow}_{\downarrow}$	s	b	b_1	b_2	b_3	b_4
d	-1.1×10^{-5}	-4.5×10^{-3}	-0.150	-0.309	-0.438	-0.5443
s		-4.55×10^{-3}	-0.150	-0.309	-0.438	-0.5443
b			-0.151	-0.309	-0.437	-0.5429
b_1				-0.279	-0.348	-0.4072
b_2					-0.180	-0.1354
b_3						$+0.1669$

and the total, for the internal quarks d, s and b are given in Table 1.

So we see that all are of the same order; but calculating the first term d contributes ~ -0.0294, s contributes ~ -0.0722 and b contributes $\sim +0.09$ resulting in total value ~ -0.0116, i.e., 39.5% of the contribution is coming from the b quark in absolute value.

Next we look into the second term of (33). The $F_1(x)$ is calculated against mass of exotic down type quark namely: $b_1 = 50\,\mathrm{GeV}$, $b_2 = 100\,\mathrm{GeV}$, $b_3 = 150\,\mathrm{GeV}$, and $b_4 = 200\,\mathrm{GeV}$, given in Table 2.

This term is also flavour diagonal and dominant contributions is coming from the large masses of exotic D. The order of contribution form d is $\sim 10^{-12}$ and that of s is $\sim 10^{-8}$ which can be neglected. The contributions from others are $b \sim +5.548 \times 10^{-5}$, $b_1 \sim -0.026$, $b_2 \sim -0.0044$, $b_3 \sim -5.386$, and $b_4 \sim +0.0108$. Thus up to b quark contribution is to the order $10^{-4}, b_1, b_2, b_3$ gradually augments negative values but considering up to b_4 the total contribution of the second term changes sign and it becomes $+0.0038$. Thus b_1, b_2, b_3 contributes constructively but the consideration of b_4 acts destructively on the amplitude.

Now for the third term which is the additional effect of the FC vertices we note the following:

(i) $F(x, y)$ is a symmetric function of x,y; $F(x, y) = F(y, x)$.

(ii) For small values of quark masses its value is negligibly small, but incorporation of exotic D quarks brings the intergenerational mixing to a comparable value but it remains negative till b_4 is considered as seen from Table 3.

We have considered here seven down quarks; four exotic quarks in addition to d, s and b. For intergenerational mixing each has six mixing terms; the contribution for each quark considered first are given in Table 4.

The cumulative value at each stage is negative; yet while considering b contribution which is positive reduces the third term a bit.

Thus we see that exotic quark mixing acts constructively on the tcZ vertex amplitude as seen from Table 5.

Table 4.

Considered up to the quark	Values of the third term	Cumulative value
d	-0.002697	$-$
s	-0.045565	-0.05179
b	$+0.015073$	-0.03672
b_1	-0.008694	-0.04541
b_2	-0.007973	-0.05338
b_3	-0.012634	-0.06602
b_4	-0.015024	-0.08104

Table 5. Values of $\mathcal{A}^{tcZ}$ in units of $\frac{g^3}{(4\pi)^2 \cos\theta_W}(\bar{c}_L \gamma_\mu t_L)$

Without FC		-0.0116
Up to second term	up to b_3	-0.0186
$-$	up to b_4	-0.0078
Up to third term	up to b_3	-0.0846
$-$	up to b_4	-0.0888

We see a large enhancement over the SM value and consideration of the second term actually matters little. Thus the experimental data may put constraints on the exotic down type quark masses and as we expect progressively increasing values the introduction of exotic down type heavy singlets slowly augments the decay rate and we get a clear testing ground to investigate the presence of Fourth Generation.

6 Conclusion

The size of the contribution to the effective tcZ vertex of the new penguin diagrams induced by a $dd'Z$ vertex is not bounded rather we see large enhancement over the SM result. Hence the experimental upper limits on $t \to cl^+l^-$ and on $t \to c\nu\bar{\nu}$ may put constraint on the exotic quark masses and may help investigation of the existence of the Fourth Generation.

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